New Mechanism For Mass Generation of Gauge Field *

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Abstract

A new mechanism for mass generation of gauge field is discussed in this paper. By introducing two sets of gauge fields and making the variations of these two sets of gauge fields compensate each other under local gauge transformations, the mass term of gauge fields is introduced into the Lagrangian without violating the local gauge symmetry of the Lagrangian. This model is a renormalizable quantum model.

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1 Introduction

Now, it is generally believed that four kinds of fundamental interactions in Nature are all gauge interactions and can be described by gauge field theory. From theoretical point of view, the principle of local gauge invariance plays a fundamental role in particle's interaction theory[1]. According to experimental results, some gauge bosons are massive [2]. A usual way to make gauge field obtain non-zero mass is to use spontaneously symmetry breaking and Higgs mechanism [3, 4, 5, 6, 7, 8, 9, 10], which is well-known in constructing the standard model [11, 12, 13]. But the Higgs mechanism is not the only mechanism that make gauge field massive. In this paper, another mechanism for the mass generation of gauge field is proposed. By introducing two sets of gauge fields, we will introduce the mass term of gauge fields without violating the local gauge symmetry of the Lagrangian. Because the Lagrangian has strict local gauge symmetry, the model is renormalizable[14].

2 The Lagrangian of The Model

Suppose that the gauge symmetry of the theory is $SU(N)$ group, $\psi(x)$ is a N-component vector in the fundamental representative space of $SU(N)$ group and the representative matrices of the generators of $SU(N)$ group are denoted by T_i ($i = 1, 2, \ldots, N^2 - 1$). They are Hermit and traceless. They satisfy:

$$[T_i \ , \ T_j] = i f_{ijk} T_k, \qquad Tr(T_i T_j) = \delta_{ij} K, \tag{2.1}$$

where f_{ijk} are structure constants of $SU(N)$ group, K is a constant which is independent of indices i and j but depends on the representation of the group. The representative matrix of a general element of the $SU(N)$ group is expressed as:

$$U = e^{-i\alpha^i T_i} \tag{2.2}$$

with α^i the real group parameters. In global gauge transformations, all α^i are independent of space-time coordinates, while in local gauge transformations, α^i are functions of space-time coordinates. U is a unitary $N \times N$ matrix.

In order to introduce the mass term of gauge fields without violating local gauge symmetry, two kinds of gauge fields $A_\mu(x)$ and $B_\mu(x)$ are needed. $A_\mu(x)$ and $B_\mu(x)$ are vectors in the canonical representative space of $SU(N)$ group. They can be expressed as linear combinations of generators :

$$A_\mu(x) = A^i_\mu(x) T_i \tag{2.3a}$$

$$B_\mu(x) = B^i_\mu(x) T_i. \tag{2.3b}$$

where $A^i_\mu(x)$ and $B^i_\mu(x)$ are component fields of gauge fields $A_\mu(x)$ and $B_\mu(x)$ respectively.

Corresponding to two kinds of gauge fields, there are two kinds of gauge covariant derivatives in the theory:

$$D_\mu = \partial_\mu - igA_\mu \tag{2.4a}$$

$$D_{b\mu} = \partial_\mu + i\alpha g B_\mu. \tag{2.4b}$$

The strengths of gauge fields $A_\mu(x)$ and $B_\mu(x)$ are defined as

$$\begin{array}{rcl} A_{\mu\nu} & = & \frac{1}{-ig}[D_\mu \ , \ D_\nu] \\ & = & \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu \ , \ A_\nu] \end{array} \tag{2.5a}$$

$$\begin{array}{rcl} B_{\mu\nu} & = & \frac{1}{i\alpha g}[D_{b\mu} \ , \ D_{b\nu}] \\ & = & \partial_\mu B_\nu - \partial_\nu B_\mu + i\alpha g[B_\mu \ , \ B_\nu]. \end{array} \tag{2.5b}$$

respectively. Similarly, $A_{\mu\nu}$ and $B_{\mu\nu}$ can also be expressed as linear combinations of generators:

$$A_{\mu\nu} = A^i_{\mu\nu} T_i \tag{2.6a}$$

$$B_{\mu\nu} = B^i_{\mu\nu} T_i. \tag{2.6b}$$

Using relations (2.1) and (2.5a,b), we can obtain

$$A^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g f^{ijk} A^j_\mu A^k_\nu \tag{2.7a}$$

$$B^i_{\mu\nu} = \partial_\mu B^i_\nu - \partial_\nu B^i_\mu - \alpha g f^{ijk} B^j_\mu B^k_\nu. \tag{2.7b}$$

The Lagrangian density of the model is

$$\begin{array}{rcl} \mathcal{L} & = & -\overline{\psi}(\gamma^\mu D_\mu + m)\psi - \frac{1}{4K} Tr(A^{\mu\nu}A_{\mu\nu}) - \frac{1}{4K} Tr(B^{\mu\nu}B_{\mu\nu}) \\ & & -\frac{\mu^2}{2K(1+\alpha^2)} Tr\left[(A^\mu + \alpha B^\mu)(A_\mu + \alpha B_\mu)\right] \end{array} \tag{2.8}$$

where α is a constant. In this paper, the space-time metric is selected as $\eta_{\mu\nu} = diag(-1,1,1,1)$, $(\mu,\nu = 0,1,2,3)$. According to relation (2.1), the above Lagrangian density $\mathcal{L}$ can be rewritten as:

$$\begin{array}{rcl} \mathcal{L} & = & -\overline{\psi}[\gamma^\mu(\partial_\mu - igA^i_\mu T_i) + m]\psi - \frac{1}{4} A^{i\mu\nu} A^i_{\mu\nu} - \frac{1}{4} B^{i\mu\nu} B^i_{\mu\nu} \\ & & -\frac{\mu^2}{2(1+\alpha^2)} (A^{i\mu} + \alpha B^{i\mu})(A^i_\mu + \alpha B^i_\mu). \end{array} \tag{2.9}$$

An obvious characteristic of the above Lagrangian is that the mass term of the gauge fields is introduced into the Lagrangian and this term does not affect the symmetry of the Lagrangian. We will prove that the above Lagrangian has strict local gauge

symmetry in the chapter 4.

Because both vector fields A_μ and B_μ are standard gauge fields, this model is a kind of gauge field model which describes gauge interactions between gauge fields and matter fields.

3 Global Gauge Symmetry and Conserved Charges

Now, let's discuss the gauge symmetry of the Lagrangian density $\mathcal{L}$. First, we will discuss the global gauge symmetry and the corresponding conserved charges. In global gauge transformation, the matter field ψ transforms as:

$$\psi \longrightarrow \psi' = U\psi, \tag{3.1}$$

where U is independent of space-time coordinates. That is

$$\partial_\mu U = 0. \tag{3.2}$$

The corresponding global gauge transformations of gauge fields A_μ and B_μ are

$$A_\mu \longrightarrow U A_\mu U^\dagger \tag{3.3a}$$

$$B_\mu \longrightarrow U B_\mu U^\dagger \tag{3.3b}$$

respectively. It is easy to prove that

$$D_\mu \longrightarrow U D_\mu U^\dagger \tag{3.4a}$$

$$D_{b\mu} \longrightarrow U D_{b\mu} U^\dagger \tag{3.4b}$$

$$A_{\mu\nu} \longrightarrow U A_{\mu\nu} U^\dagger \tag{3.5a}$$

$$B_{\mu\nu} \longrightarrow U B_{\mu\nu} U^\dagger \tag{3.5b}$$

Using all the above transformation relations, it can be strictly proved that all terms in eq(2.8) are gauge invariant. So, the whole Lagrangian density has global gauge symmetry.

Let α^i in eq(2.2) be the first order infinitesimal parameters, then, in the first order approximation, the transformation matrix U can be rewritten as:

$$U \approx 1 - i\alpha^i T^i. \tag{3.6}$$

The first order infinitesimal variations of fields ψ, $\overline{\psi}$, A_μ and B_μ are

$$\delta\psi = -i\alpha^i T^i \psi \tag{3.7a}$$

$$\delta\overline{\psi} = i\alpha^i \overline{\psi} T^i \tag{3.7b}$$

$$\delta A_\mu = \alpha^i f^{ijk} A^j_\mu T^k \tag{3.8a}$$

$$\delta B_\mu = \alpha^i f^{ijk} B^j_\mu T^k \tag{3.8b}$$

respectively. From eqs(3.8a,b) and eqs(2.3a,b), we can obtain

$$\delta A^k_\mu = \alpha^i f^{ijk} A^j_\mu \tag{3.9a}$$

$$\delta B^k_\mu = \alpha^i f^{ijk} B^j_\mu . \tag{3.9b}$$

The first order variation of the Lagrangian density is

$$\begin{array}{rcl} \delta\mathcal{L} & = & \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial \partial_\mu \psi} \delta\psi + \delta\overline{\psi} \frac{\partial \mathcal{L}}{\partial \partial_\mu \overline{\psi}} + \frac{\partial \mathcal{L}}{\partial \partial_\mu A^k_\nu} \delta A^k_\nu + \frac{\partial \mathcal{L}}{\partial \partial_\mu B^k_\nu} \delta B^k_\nu \right) \\ & = & \alpha^i \partial^\mu J^i_\mu , \end{array} \tag{3.10}$$

where

$$J^i_\mu = i\overline{\psi}\gamma_\mu T^i \psi - f^{ijk} A^{j\nu} A^k_{\mu\nu} - f^{ijk} B^{j\nu} B^k_{\mu\nu} . \tag{3.11}$$

The conserved current can also be written as

$$\begin{array}{rcl} J_\mu & = & i\overline{\psi}\gamma_\mu T^i \psi T^i + i[A^\nu \ , \ A_{\mu\nu}] + i[B^\nu \ , \ B_{\mu\nu}] \\ & = & J^i_\mu T^i . \end{array} \tag{3.12}$$

Because the Lagrangian density $\mathcal{L}$ has global gauge symmetry, the variation of $\mathcal{L}$ under global gauge transformations vanishes. That is

$$\delta\mathcal{L} = 0. \tag{3.13}$$

Because α^i are arbitrary global parameters, from eq(3.10), we can obtain the following conservation equation:

$$\partial^\mu J^i_\mu = 0. \tag{3.14}$$

The corresponding conserved charges are

$$\begin{array}{rcl} Q^i & = & \int d^3x J^{i0} \\ & = & \int d^3x (\psi^\dagger T^i \psi + [A_j \ , \ A^{j0}]^i + [B_j \ , \ B^{j0}]^i). \end{array} \tag{3.15}$$

After quantization, Q^i are generators of gauge transformation. An important feature of the above relation is that no matter what the value of parameter α is, gauge fields A_μ and B_μ contribute the same terms to the conserved currents and conserved charges.

4 Local Gauge Symmetry

If U in eq(3.1) depends on space-time coordinates, the transformation of eq(3.1) is a local $SU(N)$ gauge transformation. In this case,

$$\partial_\mu U \neq 0 \ , \quad \partial_\mu \alpha^i \neq 0 \tag{4.1}$$

The corresponding transformations of gauge fields A_μ and B_μ are

$$A_\mu \longrightarrow U A_\mu U^\dagger - \frac{1}{ig} U \partial_\mu U^\dagger \tag{4.2a}$$

$$B_\mu \longrightarrow U B_\mu U^\dagger + \frac{1}{i\alpha g} U \partial_\mu U^\dagger \tag{4.2b}$$

respectively.

Using above transformation relations, it is easy to prove that

$$D_\mu \longrightarrow U D_\mu U^\dagger \tag{4.3a}$$

$$D_{b\mu} \longrightarrow U D_{b\mu} U^\dagger . \tag{4.3b}$$

Therefore,

$$A_{\mu\nu} \longrightarrow U A_{\mu\nu} U^\dagger \tag{4.4a}$$

$$B_{\mu\nu} \longrightarrow U B_{\mu\nu} U^\dagger \tag{4.4b}$$

$$D_\mu \psi \longrightarrow U D_\mu \psi \tag{4.5}$$

$$A_\mu + \alpha B_\mu \longrightarrow U (A_\mu + \alpha B_\mu) U^\dagger \tag{4.6}$$

It can be strictly proved that the Lagrangian density $\mathcal{L}$ defined by eq(2.8) is invariant under the above local $SU(N)$ gauge transformations. Therefore the model has strict local gauge symmetry.

An obvious characteristics of this gauge field theory is that two different gauge fields A_μ and B_μ which correspond to one gauge symmetry are introduced into the theory. From eq(4.2a,b), we know that both gauge fields A_μ and B_μ are standard gauge fields. But, they have different roles in theory. It is known that, in Yang-Mills theory, gauge field can be regarded as gauge compensatory field of matter fields. In other words, if there were no gauge field, though the Lagrangian could have global gauge symmetry, it would have no local gauge symmetry. In order to make the Lagrangian have local gauge symmetry, we must introduce gauge field and make the variation of gauge field under local gauge transformations compensate the variation of the kinematical terms of matter fields. So, the form of local gauge transformation

of gauge field is determined by the form of local gauge transformation of matter fields. Similar case holds in the gauge field theory which is discussed in this paper: gauge field A_μ can regarded as gauge compensatory field of matter fields and gauge field B_μ can be regarded as gauge compensatory field of gauge field A_μ. Therefore, in this gauge field theory, the form of the local gauge transformation of gauge field A_μ is determined by the form of local gauge transformation of matter fields, and the form of local gauge transformation of gauge field B_μ is determined by the form of local gauge transformation of gauge field A_μ. And because of the compensation of gauge field B_μ, the mass term of gauge field can be introduced into the Lagrangian without violating its local gauge symmetry.

5 The Masses of Gauge Fields

The mass term of gauge fields can be written as:

$$(A^\mu \ , \ B^\mu) M \begin{pmatrix} A_\mu \\ B_\mu \end{pmatrix}. \tag{5.1}$$

where M is the mass matrix:

$$M = \frac{1}{1+\alpha^2} \begin{pmatrix} \mu^2 & \alpha\mu^2 \\ \alpha\mu^2 & \alpha^2\mu^2 \end{pmatrix}. \tag{5.2}$$

Generally speaking, physical particles generated from gauge interactions are eigenvectors of mass matrix and the corresponding masses of these particles are eigenvalues of mass matrix. M has two eigenvalues, they are

$$m_1^2 = \mu^2 \quad , \quad m_2^2 = 0. \tag{5.3}$$

The corresponding eigenvectors are

$$\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \quad , \quad \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}, \tag{5.4}$$

where,

$$\cos\theta = \frac{1}{\sqrt{1+\alpha^2}} \quad , \quad \sin\theta = \frac{\alpha}{\sqrt{1+\alpha^2}}. \tag{5.5}$$

Define

$$C_\mu = \cos\theta A_\mu + \sin\theta B_\mu \tag{5.6a}$$

$$F_\mu = -\sin\theta A_\mu + \cos\theta B_\mu. \tag{5.6b}$$

It is easy to see that C_μ and F_μ are eigenstates of mass matrix, they describe those particles generated from gauge interactions. The inverse transformations of (5.6a,b) are

$$A_\mu = \cos\theta C_\mu - \sin\theta F_\mu \tag{5.7a}$$

$$B_\mu = \sin\theta C_\mu + \cos\theta F_\mu . \tag{5.7b}$$

Then the Lagrangian density $\mathcal{L}$ given by (2.9) changes into:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_I , \tag{5.8}$$

where

$$\mathcal{L}_0 = -\overline{\psi}(\gamma^\mu \partial_\mu + m)\psi - \frac{1}{4} C_0^{i\mu\nu} C_{0\mu\nu}^i - \frac{1}{4} F_0^{i\mu\nu} F_{0\mu\nu}^i - \frac{\mu^2}{2} C^{i\mu} C_\mu^i . \tag{5.9a}$$

$$\begin{aligned}
\mathcal{L}_I \;=\; & ig\overline{\psi}\gamma^\mu(\cos\theta C_\mu - \sin\theta F_\mu)\psi \\
& -\tfrac{\cos2\theta}{2\cos\theta} g f^{ijk} C_0^{i\mu\nu} C_\mu^j C_\nu^k + \tfrac{\sin\theta}{2} g f^{ijk} F_0^{i\mu\nu} F_\mu^j F_\nu^k \\
& +\tfrac{\sin\theta}{2} g f^{ijk} F_0^{i\mu\nu} C_\mu^j C_\nu^k + g\sin\theta f^{ijk} C_0^{i\mu\nu} C_\mu^j F_\nu^k \\
& -\tfrac{1-\frac{3}{4}\sin^2 2\theta}{4\cos^2\theta} g^2 f^{ijk} f^{ilm} C_\mu^j C_\nu^k C^{l\mu} C^{m\nu} \\
& -\tfrac{\sin^2\theta}{4} g^2 f^{ijk} f^{ilm} F_\mu^j F_\nu^k F^{l\mu} F^{m\nu} + g^2 \mathrm{tg}\theta \cos2\theta f^{ijk} f^{ilm} C_\mu^j C_\nu^k C^{l\mu} F^{m\nu} \\
& -\tfrac{\sin^2\theta}{2} g^2 f^{ijk} f^{ilm} (C_\mu^j C_\nu^k F^{l\mu} F^{m\nu} + C_\mu^j F_\nu^k F^{l\mu} C^{m\nu} + C_\mu^j F_\nu^k C^{l\mu} F^{m\nu}).
\end{aligned} \tag{5.9b}$$

In the above relations, we have used the following simplified notations:

$$C_{0\mu\nu}^i = \partial_\mu C_\nu^i - \partial_\nu C_\mu^i \tag{5.10a}$$

$$F_{0\mu\nu}^i = \partial_\mu F_\nu^i - \partial_\nu F_\mu^i \tag{5.10b}$$

From eq(5.9a), it is easy to see that the mass of field C_μ is μ and the mass of gauge field F_μ is zero. That is

$$m_c = \mu \quad , \quad m_F = 0. \tag{5.11}$$

Transformations (5.7a,b) are pure algebraic operations which do not affect the gauge symmetry of the Lagrangian. They can be regarded as redefinitions of gauge fields. The local gauge symmetry of the Lagrangian is still strictly preserved after field transformations. In another words, the symmetry of the Lagrangian before transformations is completely the same as the symmetry of the Lagrangian after transformations. In fact, we do not introduce any kind of symmetry breaking in the

whole paper.

Fields C_μ and F_μ are linear combinations of gauge fields A_μ and B_μ, so the forms of local gauge transformations of fields C_μ and F_μ are determined by the forms of local gauge transformations of gauge fields A_μ and B_μ. Because C_μ and F_μ consist of gauge fields A_μ and B_μ and transmit gauge interactions between matter fields, for the sake of simplicity, we also call them gauge field, just as we call $W^\pm$ and Z^0 gauge fields in electroweak model. Therefor, two different kinds of force-transmitting vector fields exist in this gauge field theory: one is massive and another is massless.

6 Equation of Motion

The Euler-Lagrange equation of motion for fermion field can be deduced from eq(5.8):

$$[\gamma^\mu(\partial_\mu - ig\cos\theta C_\mu + ig\sin\theta F_\mu) + m]\psi = 0. \tag{6.1}$$

If we deduce the Euler-Lagrange equations of motion of gauge fields from eq(5.8), we will obtain very complicated expressions. For the sake of simplicity, we deduce the equations of motion of gauge fields from eq(2.8). In this case, the equations of motion of gauge fields A_μ and B_μ are:

$$D^\mu A_{\mu\nu} - \frac{\mu^2}{1+\alpha^2}(A_\nu + \alpha B_\nu) = ig\overline{\psi}\gamma_\nu T^i \psi T^i \tag{6.2a}$$

$$D_b^\mu B_{\mu\nu} - \frac{\alpha\mu^2}{1+\alpha^2}(A_\nu + \alpha B_\nu) = 0 \tag{6.2b}$$

respectively. In the above relations, we have used two simplified notations:

$$D^\mu A_{\mu\nu} = [D^\mu \ , \ A_{\mu\nu}] \tag{6.3a}$$

$$D_b^\mu B_{\mu\nu} = [D_b^\mu \ , \ B_{\mu\nu}]. \tag{6.3b}$$

Eqs(6.2a,b) can be expressed in terms of component fields A^i_μ and B^i_μ:

$$\partial^\mu A^i_{\mu\nu} - \frac{\mu^2}{1+\alpha^2}(A^i_\nu + \alpha B^i_\nu) = ig\overline{\psi}\gamma_\nu T^i \psi + g f^{ijk} A^j_{\mu\nu} A^{k\nu} \tag{6.4a}$$

$$\partial^\mu B^i_{\mu\nu} - \frac{\alpha\mu^2}{1+\alpha^2}(A^i_\nu + \alpha B^i_\nu) = -\alpha g f^{ijk} B^j_{\mu\nu} B^{k\nu} \tag{6.4b}$$

The equations of motion for gauge fields C_μ and F_μ can be easily obtained from eqs(6.4a,b). In other words, cos$\theta\cdot$ (6.4a) – sin$\theta\cdot$(6.4b) gives the equation of motion for gauge field C_μ, and – sin$\theta\cdot$ (6.4a) + cos$\theta\cdot$(6.4b) gives the equation of motion for gauge field F_μ.

From eq(6.2a) or (6.2b), we can obtain a supplementary condition. Using eq(6.1), we can prove that

$$[D^\lambda \ , \ \ - ig\overline{\psi}\gamma_\lambda T^i \psi T^i] = 0. \tag{6.5}$$

Let D^ν act on eq(6.2a) from the left, and let D_b^ν act on eq(6.2b) from the left, applying eq(5.4) and the following two identities:

$$[D^\lambda \ , \ [D^\nu \ , \ A_{\nu\lambda}]] = 0 \tag{6.6a}$$

$$[D_b^\lambda \ , \ [D_b^\nu \ , \ B_{\nu\lambda}]] = 0, \tag{6.6b}$$

we can obtain the following two equations

$$[D^\nu \ , \ A_\nu + \alpha B_\nu] = 0 \tag{6.7a}$$

$$[D_b^\nu \ , \ A_\nu + \alpha B_\nu] = 0 \tag{6.7b}$$

respectively. These two equations are essentially the same, they give a supplementary condition. If we expressed eqs(6.7a,b) in terms of component fields, these two equations will give the same expression:

$$\partial^\nu (A_\nu^i + \alpha B_\nu^i) + \alpha g f^{ijk} A_\nu^j B^{k\nu} = 0. \tag{6.8}$$

When $\nu = 0$, eqs(6.4a,b) don't give dynamical equations for gauge fields, because they contain no time derivative terms. They are just constrains. Originally, gauge fields A_μ^i and B_μ^i have $8(N^2-1)$ degrees of freedom, but they satisfy $2(N^2-1)$constrains and have (N^2-1) gauge degrees of freedom, therefore, gauge fields A_μ^i and B_μ^i have $5(N^2-1)$ independent dynamical degrees of freedom altogether. This result coincides with our experience: a massive vector field has 3 independent degrees of freedom and a massless vector field has 2 independent degrees of freedom.

7 The Case That Matter Fields Are Scalar Fields

In the above discussions, matter fields are spinor fields. Now, let's consider the case that matter fields are scalar fields. Suppose that there are N scalar fields

$\varphi_l(x)$ $(l = 1, 2, \cdots N)$ which form a multiplet of matter fields:

$$\varphi(x) = \begin{pmatrix} \varphi_1(x) \\ \varphi_2(x) \\ \vdots \\ \varphi_N(x) \end{pmatrix} \tag{7.1}$$

All $\varphi(x)$ form a fundamental representative space of $SU(N)$ group. In $SU(N)$ gauge transformation, $\varphi(x)$ transforms as :

$$\varphi(x) \longrightarrow \varphi'(x) = U\varphi(x) \tag{7.2}$$

The Lagrangian density is

$$\begin{array}{rcl} \mathcal{L} & = & -[(\partial_\mu - igA_\mu)\varphi]^+(\partial^\mu - igA^\mu)\varphi - V(\varphi) \\ & & -\frac{1}{4K}Tr(A^{\mu\nu}A_{\mu\nu}) - \frac{1}{4K}Tr(B^{\mu\nu}B_{\mu\nu}) \\ & & -\frac{\mu^2}{2K(1+\alpha^2)}Tr\left[(A^\mu + \alpha B^\mu)(A_\mu + \alpha B_\mu)\right] \end{array} \tag{7.3}$$

The above Lagrangian density can be expressed in terms of component fields :

$$\begin{array}{rcl} \mathcal{L} & = & -[(\partial_\mu - igA^i_\mu T_i)\varphi]^+(\partial^\mu - igA^{i\mu}T_i)\varphi - V(\varphi) \\ & & -\frac{1}{4}A^{i\mu\nu}A^i_{\mu\nu} - \frac{1}{4}B^{i\mu\nu}B^i_{\mu\nu} \\ & & -\frac{\mu^2}{2(1+\alpha^2)}(A^{i\mu} + \alpha B^{i\mu})(A^i_\mu + \alpha B^i_\mu) \end{array} \tag{7.4}$$

The general form for $V(\varphi)$ which is renormalizable and gauge invariant is

$$V(\varphi) = m^2\varphi^+\varphi + \lambda(\varphi^+\varphi)^2. \tag{7.5}$$

It is easy to prove that the Lagrangian density $\mathcal{L}$ defined by eq(7.3) has local $SU(N)$ gauge symmetry. The Euler-Lagrange equation of motion for scalar field φ is:

$$(\partial^\mu - igA^\mu)(\partial_\mu - igA_\mu)\varphi - m^2\varphi - 2\lambda\varphi(\varphi^+\varphi)^2 = 0 \tag{7.6}$$

If $N^2 - 1$ scalar fields $\varphi_l(x)$ $(l = 1, 2, \cdots N^2 - 1)$ form a multiplet of matter fields

$$\varphi(x) = \varphi_l(x)T_l, \tag{7.7}$$

then, the gauge transformation of $\varphi(x)$ is

$$\varphi(x) \longrightarrow \varphi'(x) = U\varphi(x)U^+. \tag{7.8}$$

All $\varphi(x)$ form a space of adjoint representation of $SU(N)$ group. In this case, the gauge covariant derivative is

$$D_\mu \varphi = \partial_\mu \varphi - ig[A_\mu \ , \ \varphi], \tag{7.9}$$

and the gauge invariant Lagrangian density $\mathcal{L}$ is

$$\begin{array}{rcl}\mathcal{L} & = & -\frac{1}{K} Tr[(D^\mu \varphi)^+ (D_\mu \varphi)] - V(\varphi) \\ & & -\frac{1}{4} A^{i\mu\nu} A^i_{\mu\nu} - \frac{1}{4} B^{i\mu\nu} B^i_{\mu\nu} \\ & & -\frac{\mu^2}{2(1+\alpha^2)} (A^{i\mu} + \alpha B^{i\mu})(A^i_\mu + \alpha B^i_\mu). \end{array} \tag{7.10}$$

8 A More General Model

In the above discussions, a gauge field model, which has strict local $SU(N)$ gauge symmetry and contains massive gauge bosons, is constructed. In the above model, only gauge field A_μ directly interacts with matter fields ψ or φ, gauge field B_μ doesn't directly interact with matter fields. But this restriction is not necessary in constructing the model. In this section, we will construct a more general gauge field model, in which both gauge fields interact with matter fields in the original Lagrangian. As an example, we only discuss the case that matter fields are spinor fields. The case that matter fields are scalar fields can be discussed similarly.

In chapter 4, we have prove that, under local gauge transformations, D_μ and $D_{b\mu}$ transform covariantly. It is easy to prove that $\cos^2\phi D_\mu + \sin^2\phi D_{b\mu}$ is the most general gauge covariant derivative which transforms covariantly under local $SU(N)$ gauge transformations

$$\cos^2\phi D_\mu + \sin^2\phi D_{b\mu} \longrightarrow U(\cos^2\phi D_\mu + \sin^2\phi D_{b\mu})U^+, \tag{8.1}$$

where ϕ is constant. If D_μ in eq(2.8) is replaced by $\cos^2\phi D_\mu + \sin^2\phi D_{b\mu}$, we can obtain the following Lagrangian:

$$\begin{array}{rcl}\mathcal{L} & = & -\overline{\psi}[\gamma^\mu(\cos^2\phi D_\mu + \sin^2\phi D_{b\mu}) + m]\psi \\ & & -\frac{1}{4K} Tr(A^{\mu\nu} A_{\mu\nu}) - \frac{1}{4K} Tr(B^{\mu\nu} B_{\mu\nu}) \\ & & -\frac{\mu^2}{2K(1+\alpha^2)} Tr\left[(A^\mu + \alpha B^\mu)(A_\mu + \alpha B_\mu)\right] \end{array} \tag{8.2}$$

Obviously, this Lagrangian has local $SU(N)$ gauge symmetry. Let $\mathcal{L}_\psi$ denote the part of the Lagrangian for fermions:

$$\mathcal{L}_\psi = -\overline{\psi}[\gamma^\mu(\cos^2\phi D_\mu + \sin^2\phi D_{b\mu}) + m]\psi. \tag{8.3}$$

Using eqs(2.4a,b), we can change $\mathcal{L}_\psi$ into

$$\mathcal{L}_\psi = -\overline{\psi}[\gamma^\mu(\partial_\mu - ig\cos^2\phi A_\mu + i\alpha g\sin^2\phi B_\mu) + m]\psi. \tag{8.4}$$

From the above Lagrangian, we know that both gauge fields A_μ and B_μ directly couple to matter field ψ. Substitute eqs(5.7a,b) into eq(8.4), we get

$$\mathcal{L}_\psi = -\overline{\psi}[\gamma^\mu(\partial_\mu - ig\frac{\cos^2\theta - \sin^2\phi}{\cos\theta}C_\mu + ig\sin\theta F_\mu) + m]\psi. \tag{8.5}$$

The equation of motion for fermion field ψ is

$$[\gamma^\mu(\partial_\mu - ig\frac{\cos^2\theta - \sin^2\phi}{\cos\theta}C_\mu + ig\sin\theta F_\mu) + m]\psi = 0. \tag{8.6}$$

The equations of motion for gauge fields A_μ and B_μ now change into:

$$D^\mu A_{\mu\nu} - \frac{\mu^2}{1+\alpha^2}(A_\nu + \alpha B_\nu) = ig\cos^2\phi\overline{\psi}\gamma_\nu T^i\psi T^i \tag{8.7a}$$

$$D_b^\mu B_{\mu\nu} - \frac{\alpha\mu^2}{1+\alpha^2}(A_\nu + \alpha B_\nu) = -i\alpha g\sin^2\phi\overline{\psi}\gamma_\nu T^i\psi T^i. \tag{8.7b}$$

If ϕ vanish, the Lagrangian density (8.2) will return to the original Lagrangian density (2.8), the equations of motion (8.7a,b) will return to eqs(6.2a,b), and eq(8.6) will return to eq(6.1). So, the model discussed in the above chapters is just a special case of the model we discuss in this chapter.

9 U(1) Case

If the symmetry of the model is U(1) group, we will obtain a U(1) gauge field model. We also use A_μ and B_μ to denote gauge fields and ψ to denote a fermion fields. In U(1) case, the strengths of gauge fields are

$$A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{9.1a}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \tag{9.1b}$$

Two gauge covariant derivatives are the same as (2.4a,b) but with different content. The Lagrangian density of the model is:

$$\begin{aligned}\mathcal{L} \;=\; & -\overline{\psi}[\gamma^\mu(\cos^2\phi D_\mu + \sin^2\phi D_{b\mu}) + m]\psi \\ & -\tfrac{1}{4}A^{\mu\nu}A_{\mu\nu} - \tfrac{1}{4}B^{\mu\nu}B_{\mu\nu} \\ & -\tfrac{\mu^2}{2(1+\alpha^2)}(A^\mu + \alpha B^\mu)(A_\mu + \alpha B_\mu)\end{aligned} \tag{9.2}$$

Local U(1) gauge transformations are

$$\psi \longrightarrow e^{-i\theta}\psi, \tag{9.3a}$$

$$A_\mu \longrightarrow A_\mu - \frac{1}{g}\partial_\mu\theta \tag{9.3b}$$

$$B_\mu \longrightarrow B_\mu + \frac{1}{\alpha g}\partial_\mu\theta. \tag{9.3c}$$

Then, $A_{\mu\nu}$, $B_{\mu\nu}$ and $A_\mu + \alpha B_\mu$ are all U(1) gauge invariant. That is

$$A_{\mu\nu} \longrightarrow A_{\mu\nu} \tag{9.4a}$$

$$B_{\mu\nu} \longrightarrow B_{\mu\nu} \tag{9.4b}$$

$$A_\mu + \alpha B_\mu \longrightarrow A_\mu + \alpha B_\mu \tag{9.4c}$$

Using all these results, it is easy to prove that the Lagrangian density given by eq(9.2) has local $U(1)$ gauge symmetry.

Substitute eqs(5.7a,b) into eq(9.2), the Lagrangian density $\mathcal{L}$ changes into

$$\begin{array}{rcl} \mathcal{L} & = & -\overline{\psi}[\gamma^\mu(\partial_\mu - ig\frac{\cos^2\theta - \sin^2\phi}{\cos\theta}C_\mu + ig\sin\theta F_\mu) + m]\psi \\ & & -\frac{1}{4}C^{\mu\nu}C_{\mu\nu} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{\mu^2}{2}C^\mu C_\mu \end{array} \tag{9.5}$$

where,

$$C_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu \tag{9.6a}$$

$$F_{\mu\nu} = \partial_\mu F_\nu - \partial_\nu F_\mu \tag{9.6b}$$

So, in this model, there is a massive Abel gauge field as well as a massless Abel gauge field. They all have gauge interactions with matter field. In this case, $U(1)$ gauge interactions is transmitted by two different kinds of gauge fields.

It is known that QED is a U(1) gauge field theory. According to the model we discuss here, we may guess that there may exist two different kinds of photon, one is massive while another is massless. And if θ is near $\pi/2$ and the mass of massive photon is large, the massless photon field couples with charged fields will be much stronger than massive photon.

10 Two limits of the model.

Now, we let's discuss two kinds of limits of this model. The first kind of limits corresponds to very small parameter α. Let

$$\alpha \longrightarrow 0, \tag{10.1}$$

then

$$\cos\theta \approx 1 \quad , \quad \sin\theta \approx 0. \tag{10.2}$$

From eqs(5.6a,b), we know that the gauge field A_μ is just gauge field C_μ and the gauge field B_μ is just the gauge field F_μ. That is

$$C_\mu \approx A_\mu \quad , \quad F_\mu \approx B_\mu. \tag{10.3}$$

In this case, the Lagrangian density (2.9) becomes

$$\begin{array}{rcl} \mathcal{L} & \approx & -\overline{\psi}[\gamma^\mu(\partial_\mu - igC^i_\mu T^i) + m]\psi \\ & & -\frac{1}{4}C^{i\mu\nu}C^i_{\mu\nu} - \frac{1}{4}F^{i\mu\nu}F^i_{\mu\nu} - \frac{\mu^2}{2}C^{i\mu}C^i_\mu. \end{array} \tag{10.4}$$

The massless gauge field do not interact with matter fields in this limit. So, the $\alpha \longrightarrow 0$ limit corresponds to the case that gauge interactions are mainly transmitted by massive gauge field. So, the above Lagrangian approximately describes those kinds of gauge interactions which are dominated by massive gauge bosons.

The second kind of limits corresponds to very big parameter α. Let

$$\alpha \longrightarrow \infty, \tag{10.5}$$

then

$$\cos\theta \approx 0 \quad , \quad \sin\theta \approx 1. \tag{10.6}$$

From eqs(5.6a,b), we know that:

$$C_\mu \approx B_\mu \quad , \quad F_\mu \approx -A_\mu. \tag{10.7}$$

Then, the Lagrangian density (2.9) becomes

$$\begin{array}{rcl} \mathcal{L} & \approx & -\overline{\psi}[\gamma^\mu(\partial_\mu + igF^i_\mu T^i) + m]\psi \\ & & -\frac{1}{4}F^{i\mu\nu}F^i_{\mu\nu} - \frac{1}{4}C^{i\mu\nu}C^i_{\mu\nu} - \frac{\mu^2}{2}C^{i\mu}C^i_\mu. \end{array} \tag{10.8}$$

In this case, massive gauge field does not directly interact with matter fields. So, this limit corresponds to the case when gauge interactions are mainly transmitted by massless gauge field.

In the particles' interaction model which describes the gauge interactions of real world, the parameter α is finite,

$$0 < \alpha < \infty. \tag{10.9}$$

In this case, both massive gauge field and massless gauge field directly interact with matter fields, and gauge interactions are transmitted by both of them.

11 The renormalizability of the theory

The renormalizability of the theory can be very strictly proved[14]. But this proof is extremely long and is not suitable to write it here. We will not discuss it in details in this paper. We only want to discuss some key problems on the renormalizability of the theory.

It is know that, according to the power counting law, a massive vector field model is not renormalizable in most case. The reason is simple. It is known that the propagator of a massive vector field usually has the following form:

$$\Delta_{F\mu\nu} = \frac{-i}{k^2 + \mu^2 - i\varepsilon}\left(g_{\mu\nu} + \frac{k_\mu k_\nu}{\mu^2}\right). \tag{11.1}$$

So, when we let

$$k \longrightarrow \infty \tag{11.2}$$

then,

$$\Delta_{F\mu\nu} \longrightarrow const. \tag{11.3}$$

In this case, there are infinite kinds of divergent Feynman diagrams. According to the power counting law, this theory is a kind of non-renormalizable theory. Though gauge field theory contains massive vector fields, it is renormalizable. The key reason is that the Lagrangian has local gauge symmetry[8].

As we have stated before, this gauge field theory has maximum local $SU(N)$ gauge symmetry. When we quantize this gauge field theory in the path integral formulation, we must select gauge conditions first [17]. In order to make gauge transformation degree of freedom completely fixed, we must select two gauge conditions simultaneously: one is for massive gauge field C_μ and another is for massless gauge field F_μ. For example, if we select temporal gauge condition for massless gauge field F_μ:

$$F_4 = 0, \tag{11.4}$$

there still exists remainder gauge transformation degree of freedom, because temporal gauge condition is unchanged under the following local gauge transformation:

$$F_\mu \longrightarrow U F_\mu U^\dagger + \frac{1}{ig\sin\theta} U \partial_\mu U^\dagger \tag{11.5}$$

where

$$\partial_t U = 0, U = U(\vec{x}). \tag{11.6}$$

In order to make this remainder gauge transformation degree of freedom completely fixed, we'd better select another gauge condition for gauge field C_μ. For example, we can select the following gauge condition for gauge field C_μ:

$$\partial^\mu C_\mu = 0. \tag{11.7}$$

If we select two gauge conditions simultaneously, when we quantize the theory in path integral formulation, there will be two gauge fixing terms in the effective Lagrangian. The effective Lagrangian can be written as:

$$\mathcal{L}_{eff} = \mathcal{L} - \frac{1}{2\alpha_1} f_1^a f_1^a - \frac{1}{2\alpha_2} f_2^a f_2^a + \overline{\eta}_1 M_{f1} \eta_1 + \overline{\eta}_2 M_{f2} \eta_2 \tag{11.8}$$

where

$$f_1^a = f_1^a(F_\mu), f_2^a = f_2^a(C_\mu) \tag{11.9}$$

If we select

$$f_2^a = \partial^\mu C_\mu^a, \tag{11.10}$$

then the propagator for massive gauge field C_μ is:

$$\Delta_{F\mu\nu}^{ab}(k) = \frac{-i\delta^{ab}}{k^2 + \mu^2 - i\varepsilon} \left(g_{\mu\nu} - (1 - \frac{1}{\alpha_2}) \frac{k_\mu k_\nu}{k^2 - \mu^2/\alpha_2} \right). \tag{11.11}$$

If we let k approach infinity, then

$$\Delta_{F\mu\nu}^{ab}(k) \sim \frac{1}{k^2}. \tag{11.12}$$

In this case, according to the power counting law, the gauge field theory which is discussed in this paper is a kind of renormalizable theory. At the same time, the local $SU(N)$ gauge symmetry will give a Ward-Takahashi identity which will eventually make the theory renormalizable. A strict proof on the renormalizability of the gauge field theory can be found in the reference [15].

In order to make the gauge field theory renormalizable, it is very important to keep the maximum local $SU(N)$ gauge symmetry of the Lagrangian. From the above discussions, we know that, in the renormalization of the gauge field theory, local gauge symmetry plays the following two important roles: 1) to make the propagator of the massive gauge bosons have the renormalizable form; 2) to give a Ward-Takahashi identity which plays a key role in the proof of the renormalizability of the gauge field theory.

12 Comments

Up to now, we know that there are two mechanisms that can make gauge field obtain non-zero mass: one is the Higgs mechanism which is well known in constructing the Standard Model; another is the mechanism discussed in this paper. In this new mechanism, the mass term of gauge field is introduced by using another set of gauge field and the mass term of gauge fields does not affect the symmetry of the Lagrangian. We can imagine the new interaction picture as: when matter fields take part in gauge interactions, they emit or absorb one kind of gauge field which is not eigenstate of mass matrix, when we detect this gauge fields in experiments, it will appear in two states which corresponds to two kinds of vector fields, one is massless and another is massive.

Though the Lagrangian of the model contains the mass term of gauge fields, the theory is renormalizable. So, we can use the mechanism to describe gauge interactions of quarks and leptons. If we apply this mechanism to electroweak interactions, we can construct an electroweak model which contains no Higgs particle.

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